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Atanassov's Intuitionistic Fuzzy Hyper-Homomorphism of the Intuitionistic Fuzzy Hyperrings Based on Intuitionistic Fuzzy Space

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Abstract

My study introduces the notation of intuitionistic fuzzy hyper-homomorphism on intuitionistic fuzzy hyperrings based on intuitionistic fuzzy spaces. It also presents the concept of an intuitionistic fuzzy normal sub-hyperrings induced by intuitionistic fuzzy subsets. Finally, we formulate some properties of the normalizer and the intuitionistic fuzzy version of the fundamental theorem of hyper-homomorphism and those hyper-isomorphism theorems.

Keywords: The intuitionistic fuzzy sets, The intuitionistic fuzzy space, The intuitionistic fuzzy homomorphism, The intuitionistic fuzzy rings, The fuzzy algebra, The hyper algebra.

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التشاكل الفائق الضبابي الحدسي وفق أتاناسوف للحلقات الفائقة الضبابية الحدسية القائمة على الفضاء الضبابي الحدسي

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الملخص

تقدم هذه الدراسة مفهوم التشاكل الفائق الضبابي الحدسي على الحلقات الفائقة الضبابية الحدسية استنادا إلى الفضاءات الضبابية الحدسية. كما تعرض مفهوم الحلقات الفائقة الجزئية الضبابية الحدسية النظامية المولّدة من المجموعات الجزئية الضبابية الحدسية. وأخيرا، تصوغ الدراسة عددا من الخصائص المتعلقة بالنظريات العادية والضبابية الفائقة، إلى جانب الصيغة الضبابية الحدسية لمبرهنة التشاكل الفائق الأساسية، وكذلك مبرهنات التشاكل الفائق.

الكلمات المفتاحية: المجموعات الضبابية الفائقة، الفضاء الضبابي الفائق، التشاكل الضبابي الفائق، الحلقات الضبابية الفائقة، الجبر الضبابي، الجبر الفائق.

1. Introduction

The concept of the intuitionistic fuzzy sets (IFSs) introduced by Atanassov (1986), such that for each element there are two functions, the membership function and the co-membership function as generaliztion of the concept of the fuzzy set. In particular, they introduced some properties and concepts of images and inverse images under fuzzy homomorphisms, as studied the fuzzy kernels as fuzzy congruence relations, and proved that the composition between the notions of the fuzzy homomorphisms is a fuzzy homomorphism. Giansiracusa and Lorscheid (2016) have contributed a theoretical addition in their work about the relation between hyperrings and fuzzy rings, their main study was to construct a categorical connection between the notion of the fuzzy rings and the hyperrings.

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Also, the concept of the intuitionistic fuzzy algebra was studied by Sharma (2011), who developed the preservation of intuitionistic fuzzy subsets under homomorphic and inverse pictures and intuitionistic fuzzy homomorphism, and generalized the application from intuitionistic fuzzy to the abstract algebra and the classical properties from the homomorphism to the intuitionistic fuzzy algebra.

Abdulmula and Salleh (2012) contributed greatly to the theory of hyperstructures by "the intuitionistic fuzzy hyperhomomorphism and intuitionistic fuzzy normal subhypergroups". Their aim is to investigate the algebraic properties of intuitionistic fuzzy hyperhomomorphisms on hypergroups. They provided the first complete analysis of the notion of the hyperhomomorphisms in the intuitionistic fuzzy environment by establishing the connections between the intuitionistic fuzzy hyperhomomorphisms and the intuitionistic fuzzy normal subhypergroups, their study was about the intuitionistic fuzzy hyperideals in semihypergroups and their structural properties were discussed in another paper in the same year. They with Davvaz (2013), introduced in their study Atanassov's intuitionistic fuzzy hyperrings based on intuitionistic fuzzy universal sets. The main aim of them was to develop the concept of the intuitionistic fuzzy hyperrings as generalization of intuitionistic fuzzy universal sets and to study their properties. They established mathematical basis of the intuitionistic fuzzy hyperrings by proving a series of important results on congruence relations, algebraic operations and intuitionistic fuzzy hyperideals.

Firouzkouhi, Amini, Cheng, Soleymani, and Davvaz (2021) recently introduced basic relations in the concept of the fuzzy hyperalgebras. Where was their work about how algebraic identities can be generalized in fuzzy hyperstructures, and extended the application of fuzzy hyperalgebra theory.

The concept of intuitionistic fuzzy soft hyperalgebras has been introduced by Bolat, Onar and Ersoy (2023). Their study revealed that the intuitionistic fuzzy hyperstructure with the soft set theory introduced the mean idea of the algebraic decision.

In a recent study Talaee, Oskooie and Davvaz (2024) studied the notion of the intuitionistic fuzzy modules with special properties of

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the intuitionistic fuzzy maps. They generalized some of the increasing importance of concept the intuitionistic fuzzy algebra.

In this paper, we introduce the concept of intuitionistic fuzzy hyperhomomorphisms of intuitionistic fuzzy hyperrings based on the intuitionistic fuzzy space, as that their basic algebraic properties are studied and their theory is developed. Some new results related to this notion such as the kernel, the image, inverse images, and preservation of intuitionistic fuzzy hyperideals are introduced. Such concepts are expected to the intuitionistic fuzzy hyperring theory and provide a foundation for further researches on generalized algebraic structures in uncertainty

2. Intuitionistic Fuzzy Hyper-Homomorphism

Here in this section, we show the notation of hyper-rings and study hyper-homomorphisms of intuitionistic fuzzy hyper-rings. Next, we generalize the notion of the fuzzy homomorphism to the concept of intuitionistic fuzzy hyper-homomorphism (IFHH) of intuitionistic fuzzy hyper-rings (IFHR) based on intuitionistic fuzzy space (IFS) as in the following;

2.1. Definition [8]

Let $\underline{F^+} = (F^+, \underline{f_{xy}}^+, \overline{f_{xy}}^+)$ and $\underline{F^*} = (F^*, \underline{f_{xy}}^*, \overline{f_{xy}}^*)$ be two intuitionistic fuzzy binary hyperoperations on the intuitionistic fuzzy space (H, I, I). We call $\langle (H, I, I), \underline{F^+}, \underline{F^*} \rangle$ an intuitionistic fuzzy hyperring if the following axioms are satisfied;

- **a.** $\langle (H, I, I), \underline{F^+} \rangle$ is an intuitionistic fuzzy hypergroup,
- **b.** $\langle (H, I, I), \underline{F^*} \rangle$ is an intuitionistic fuzzy semi hypergroup,
- **c.** The distributive laws. That is,

$$(x, I, I)\underline{F^*}\ \big((y, I\ I)\underline{F^+}\ (z, I, I)\big) = \big((x, I, I)\underline{F^*}\,(y, I, I)\big)\,\underline{F^+}\ \big((x, I, I)\underline{F^*}\,(z, I, I)\big),$$

$$\big((x, I, I)\underline{F^+}\,(y, I, I)\big)\ \underline{F^*}(z, I, I) = \big((x, I, I)\underline{F^*}\,(z, I, I)\big)\,\underline{F^+}\ \big((y, I, I)\ \underline{F^*}(z, I, I)\big),$$

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for every intuitionistic fuzzy elements $(x, I, I), (y, I, I)$ and (z, I, I) of (H, I, I).

- **Example**

Let the space $\mathcal{R} = \{0\,, a\,, b\,, c\}$, define two intuitionistic fuzzy binary hyperoperations $\underline{F}^{+}$and $\underline{F}^{*}$ on the intuitionistic fuzzy space $(\mathcal{R}, I, I)$ as following;

Table 1. The first fuzzy binary Operation

$\underline{F}^{+}$	0	a	b	c
0	$\{0\}$	$\{a\}$	$\{b\}$	$\{c\}$
a	$\{a\}$	$\{0, a\}$	$\{a\ b\}$	$\{a\ c\}$
b	$\{b\}$	$\{a, b\}$	$\{0, c\}$	$\{a, b\,, c\}$
c	$\{c\}$	$\{a, c\}$	$\{a, b, c\}$	$\{b, 0\}$

Table 2. The second fuzzy binary Operation

$\underline{F}^{*}$	0	a	b	c
0	$\{0\}$	$\{0\}$	$\{0\}$	$\{0\}$
a	$\{0\}$	$\{a\}$	$\{b\}$	$\{c\}$
b	$\{0\}$	$\{b\}$	$\{c\}$	$\{a\}$
c	$\{0\}$	$\{c\}$	$\{a\}$	$\{b\}$

where the membership functions $\underline{f^{+}}_{xy}(r, s)$, $\underline{f^{*}}_{xy}\ (r\ ,\ s)$, from $I \times I$ into I as:

$$\underline{f^{+}}_{xy}(r, s) = \begin{cases} \dfrac{(r * s)}{1 - (r * s)} & , r \vee s < \dfrac{1}{2} \\ 1 & , r \vee s \geq \dfrac{1}{2} \end{cases}$$

$$\underline{f^{*}}_{xy}(r, s) = \begin{cases} \dfrac{(r * s)}{\max(r, s, \frac{1}{2})} & ,\ r \wedge s < \dfrac{3}{4} \\ \max(r, s) & , r \wedge s \geq \dfrac{3}{4} \end{cases}$$

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and the conon-membership functions are $\underline{f^{+}}_{xy}(r,s): I \times I \longrightarrow I \max(r,s)$,
$\underline{f^{*}}_{xy}(r,s): I \times I \longrightarrow I = \min(r,s)$. Now must proof that $\langle (R,I,I), \underline{F}^{+}, \underline{F}^{*} \rangle$ is an intuitionistic fuzzy hyperring,

I. we must show $\langle (R,I,I), \underline{F}^{+} \rangle$ is intuitionistic fuzzy hypergroup, if ({a} , I ,I) , ({b}, I , I) and ({c} ,I ,I,) are intuitionistic fuzzy elements in the intuitionistic fuzzy hypergroup $\langle (R,I,I), \underline{F}^{+} \rangle$, then we

[({a} , I ,I) $\underline{F}^{+}$ ({b}, I , I)] $\underline{F}^{+}$ ({c} ,I ,I,) = ({a} , I ,I) $\underline{F}^{+}$[({b}, I , I) $\underline{F}^{+}$
({c} ,I ,I,) = ({a, b, c} ,I ,I),
({0},I, I) $\underline{F}^{+}$ ({R ,I ,I) = ({0},I, I) $\underline{F}^{+}$ {(t ,I ,I) ; t ∈ R } = (R ,I ,I) = (R ,I ,I) $\underline{F}^{+}$ ({0},I, I), then, $\langle (R,I,I), \underline{F}^{+} \rangle$ is intuitionistic fuzzy hypergroup,

II. we must show $\langle (R,I,I), \underline{F}^{*} \rangle$ is an intuitionistic fuzzy semihypergroup, if ({a} , I ,I) , ({b}, I , I), and ({c} ,I ,I,) are intuitionistic fuzzy elements in the intuitionistic fuzzy hypergroup $\langle (R,I,I), \underline{F}^{*} \rangle$, then

[({0} , I ,I) $\underline{F}^{*}$ ({b}, I , I)] $\underline{F}^{*}$ ({c} ,I ,I,) = ({b} , I , I) $\underline{F}^{*}$ ({c} ,I ,I,) = ({a } , I . I),
({0}, I ,I) $\underline{F}^{*}$[({b}, I , I) $\underline{F}^{*}$({c} , I, I,)] = ({0}, I , I) $\underline{F}^{*}$({a } ,I ,I,) = ({a } , I . I),
then from (1) , (2), we get the intuitionistic fuzzy hyperring $\langle (R,I,I), \underline{F}^{+}, \underline{F}^{*} \rangle$.

- **Theorem**

Let $S = \{(X, \underline{S_x}, \overline{S}_x)$: x∈ $S_0\}$ be an intuitionistic fuzzy subspace of the intuitionistic fuzzy space (R,I,I). Then (S, $\underline{F}$) is an intuitionistic fuzzy subhypergroup of the intuitionistic fuzzy hypergroup (intuitionistic fuzzy $H_v - group$) $((H,I,I), \underline{F})$ *If;:*

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I. (S_0, F) is an ordinary subhypergroup of hypergroup (H, F) ,

II. $\underline{f}_{xy}(\underline{s}_x, \bar{s}_y) = \underline{s}_{xFy}$ and $\overline{f}_{xy}(\underline{s}_x, \bar{s}_y) = \overline{f}_{xFy}$ $for\ all\ x, y \in S_0,$

2.2. Definition

If $\Omega = (\psi, \underline{\psi_x}, \overline{\psi_x}) : \langle (R, I, I), \underline{F}^+, \underline{F}^* \rangle \longrightarrow \langle (\acute{R}, I, I), \underline{\tilde{F}}^+, \underline{\breve{F}}^* \rangle$, where (R, *I*, *I*) and $(\acute{R}, I, I)$ are intuitionistic fuzzy spaces and $\langle (R, I, I), \underline{F}^+, \underline{F}^* \rangle$, $\langle (\acute{R}, I, I), \underline{\tilde{F}}^+, \underline{\breve{F}}^* \rangle$ are two (IFHRs) "intuitionistic fuzzy hyperrings", then **Ω** is an intuitionistic fuzzy hyper-homomorphism (IFHH) on intuitionistic fuzzy hyper-rings as an intuitionistic fuzzy functions $\underline{F}^+, \underline{\tilde{F}}^+$ having two onto comembership functions
$\underline{F}^+, \underline{\tilde{F}}^+ = (\underline{F}^+ , \underline{\boldsymbol{f}_{r\acute{r}}}, \overline{\boldsymbol{f}_{r\acute{r}}})$, $\underline{\boldsymbol{f}_{r\acute{r}}}, \overline{\boldsymbol{f}_{r\acute{r}}} : I \times I \rightarrow I$, and co-non-membership functions,
$\underline{F}^*, \underline{\tilde{F}}^* = (\underline{F}^* , \underline{\boldsymbol{f}_{r\acute{r}}}, \overline{\boldsymbol{f}_{r\acute{r}}})$, $\underline{\boldsymbol{f}_{r\acute{r}}}, \overline{\boldsymbol{f}_{r\acute{r}}} : I \times I \rightarrow I$, where these the functions hold the following,

$$\Omega = (\psi, \underline{\psi_x}, \overline{\psi_x}) : \langle (R, I, I), \underline{F}^+, \underline{F}^* \rangle \longrightarrow \langle\langle (\acute{R}, I, I), \underline{\tilde{F}}^+, \underline{\breve{F}}^* \rangle,$$

$$\Omega\left((r, I, I)\, \underline{F}^+(r, I, I), (r, I, I)\underline{F}^*\, (r, I, I)\right) =$$

$$\Omega\left((r\underline{F}^+\acute{r}, r\underline{\acute{F}}^*r\ ), \underline{f_{r\acute{r}}}\ (I, I), \overline{f_{r\acute{r}}}\ (I, I)\right)$$

$$= \left(\psi(r, \acute{r}), \underline{\psi_x}\ (I, I), \overline{\psi_x}\ (I, I)\right)$$

- **Remark**

1) We can say that the intuitionistic fuzzy hyperrings $\langle (R, I, I), \underline{F}^+, \underline{F}^* \rangle,\ and\ \langle (\acute{R}, I, I), \underline{\tilde{F}}^+, \underline{\breve{F}}^* \rangle$ are intuitionistic fuzzy hyper-homomorphism hyperrings under $\Omega\left(\psi, \underline{\psi_x}, \overline{\psi_x}\right)$.

2) We call $\Omega(\psi, \underline{\psi_x}, \overline{\psi_x})$ an intuitionistic fuzzy hyper-homomorphism if $\psi : R \rightarrow \acute{R}$.

3) The intuitionistic fuzzy hyper-homomorphism **Ω** is an intuitionistic fuzzy isomorphism hyperrings

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and will be denoted by $\langle (R, I, I), \underline{F}^{+}, \underline{F}^{*} \rangle \cong \langle (\acute{R}, I, I), \tilde{\underline{F}}^{+}, \check{\underline{F}}^{*} \rangle$, the action of (IFHH) $\Omega(\psi, \underline{\psi_x}, \overline{\psi_x})$ of any two intuitionistic fuzzy elements (r, I, I), $(\acute{r}, I, I) \in (R, I, I)$ is as follows;

I. $\Omega\left((r, I, I)\, \underline{F}^{+}(r, I, I)\right) = \Omega\left((\underline{F}^{+}(r, \acute{r}), \underline{f_{r\acute{r}}}(I \times I), \overline{f_{r\acute{r}}}(I \times I)\right) = \Omega\left((r\underline{F}^{+}\acute{r}), \underline{f_{r\acute{r}}}(I, I), \overline{f_{r\acute{r}}}(I, I)\right)$
$= \left(\psi(r, \acute{r}), \underline{\psi_x}(I, I), \overline{\psi_x}(I, I)\right)$,

II. $\Omega\left((r, I, I)\underline{F}^{*}(r, I, I)\right) = \Omega\left((\underline{F}^{\acute{*}}(r, \acute{r}), \underline{f_{r\acute{r}}}(I \times I), \overline{f_{r\acute{r}}}(I \times I)\right)$
$= \Omega\left((\underline{r\acute{F}^{*}r}), \underline{f_{r\acute{r}}}(I, I), \overline{f_{r\acute{r}}}(I, I)\right)$
$= \left(\psi(r, \acute{r}), \underline{\psi_x}(I, I), \overline{\psi_x}(I, I)\right)$

- **Remark**

An intuitionistic fuzzy hyper-homomorphism **Ω** is an intuitionistic fuzzy hyperhomomorphism of [1], if it holds; $\Omega\left(\psi, \underline{\psi_x}, \overline{\psi_x}\right): \langle (R, I, I), \underline{F}^{+}, \underline{F}^{*} \rangle \to \langle (\acute{R}, I, I), \tilde{\underline{F}}^{+}, \check{\underline{F}}^{*} \rangle$, where,

I. $\underline{F}^{+}: \langle (R, I, I) \rangle \times \langle (R, I, I) \rangle \to (R, I, I) \subseteq \mathrm{P}^{*}\langle (R, I, I) \rangle$,

II. $\underline{F}^{*}: \langle (\acute{R}, I, I) \rangle \times \langle (\acute{R}, I, I) \rangle \to (\acute{R}, I, I) \subseteq \mathrm{P}^{*}\langle (\acute{R}, I, I) \rangle$,

from the above argument and definition, we have the next theorem, which relates IFHH on rings as the following;

- **Theorem**

The correspondence intuitionistic fuzzy hyper-homomorphism hyperrings $[\langle (R, I, I), \underline{F}^{+}, \underline{F}^{*} \rangle, \langle (\acute{R}, I, I), \tilde{\underline{F}}^{+}, \check{\underline{F}}^{*} \rangle]$ of the fuzzy hyperrings $[\langle (R, I), \underline{F}^{+}, \underline{F}^{*} \rangle, \langle (\acute{R}, I), \tilde{\underline{F}}^{+}, \check{\underline{F}}^{*} \rangle]$ are ordinary fuzzy hyper- homomorphic.

- **Proof**

If we have two intuitionistic fuzzy hyper-homomorphism hyperrings $\langle (R, I, I), \underline{F}^{+}, \underline{F}^{*} \rangle\ and\ \langle (\acute{R}, I, I), \tilde{\underline{F}}^{+}, \check{\underline{F}}^{*} \rangle$ under

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intuitionistic fuzzy hyper-homomorphism $\Omega\left(\psi, \underline{\psi_x}, \overline{\psi_x}\right)$ which corresponds to fuzzy hyperrings $\langle (R, I), \underline{F}^+, \underline{F}^* \rangle$ *and* $\langle (\acute{R}, I), \underline{\tilde{F}}^+, \underline{\check{F}}^* \rangle$, then by remark that the correspondences (r, I, I) $\leftrightarrow$ (r, I) ϵ (R, I, I), and ($\acute{r}, I, I$) $\leftrightarrow$ ($\acute{r}, I$) ϵ ($\acute{R}, I, I$), as using the formulation obtained in $\Omega\left(\psi, \underline{\psi_x}, \overline{\psi_x}\right)$, we get $\Omega\ (r_1\ \underline{\tilde{F}}^+\ r_2) = \underline{\psi_x}\,(r_1)\ \underline{\tilde{F}}^+ \underline{\psi_x}\,(r_2) = \acute{r}_1\ \underline{\tilde{F}}^+\ \acute{r}_2$ and $\Omega\ (r_1\ \underline{\check{F}}^*\ r_2) = \underline{\psi_x}\,(r_1) \underline{\check{F}}^*\ \underline{\psi_x}\,(r_2) = \acute{r}_1\ \underline{\check{F}}^*\ \acute{r}_2$.

2.3. Remark

Let $\langle (R, I, I), \underline{F}^+, \underline{F}^* \rangle$ *and* $\langle (\acute{R}, I, I), \underline{\tilde{F}}^+, \underline{\check{F}}^* \rangle$ two intuitionistic fuzzy hyper-homomorphisms hyperrings, then there exists a correspondence between ordinary hyperrings (R, F).

In the next lemma, we introduce two main properties of IFHHr (intuitionistic fuzzy hyper-homomorphism hyperrings and the ordinary homomorphism on two fuzzy rings as the following;

- **Lemma**

Let $\Omega : \langle (R, I, I), \underline{F}^+, \underline{F}^* \rangle \longrightarrow \langle (\acute{R}, I, I), \underline{\tilde{F}}^+, \underline{\check{F}}^* \rangle$ be an intuitionistic fuzzy hyper-homomorphism on two intuitionistic fuzzy hyperrings $\langle (R, I, I), \underline{F}^+, \underline{F}^* \rangle \cong \langle (\acute{R}, I, I), \underline{\tilde{F}}^+, \underline{\check{F}}^* \rangle$, and having intuitionistic fuzzy identities **(**e, I, I) and **(**$\acute{e}$, I, I) respectively, then the following hold;

1. Ω (e, I, I) = **(**$\acute{e}, I, I$).
2. Ω (r^{-1}, I, I) = (($\Omega\ (r, I, I\ ))^{-1}$.

- **Proof**

By using the properties of intuitionistic fuzzy hyper-homomorphism of intuitionistic fuzzy hyperrings, we can get the proof.

If an intuitionistic fuzzy subspace S of the intuitionistic fuzzy space (R, I, I), is as S = $\{(r\ ,\ \underline{s_r}\ , \overline{s_r}\ ) : r\ \in\ s_0\}$, and we have intuitionistic fuzzy hyper-homomorphism **Ω** of two intuitionistic fuzzy hyperrings $\langle (R, I, I), \underline{F}^+, \underline{F}^* \rangle$ *and* $\langle (\acute{R}, I, I), \underline{\tilde{F}}^+, \underline{\check{F}}^* \rangle$, then the image for any the intuitionistic fuzzy sub-hyperring $\langle S, \underline{F}^+, \underline{F}^* \rangle$ of the intuitionistic fuzzy hyperring $\langle (R, I, I), \underline{F}^+, \underline{F}^* \rangle$ under **Ω** is an intuitionistic fuzzy sub-hyperring $\langle \acute{S}, \underline{\acute{F}^+}, \underline{F}^* \rangle$ of

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$\langle(\acute{R}, I, I), \underline{\tilde{F}}^{+}, \underline{\check{F}}^{*}\rangle$, if it satisfies the following; $\Omega : (r\,, \underline{s_r}\,, \overline{s_r}\,) \rightarrow (\acute{r}\,, \underline{s_{\acute{r}}}\,, \overline{s_{\acute{r}}}\,) = \left(\psi(r_1, r_2), \underline{\psi_x}(\underline{s_{r_1}}, \underline{s_{r_2}})\,, \overline{\psi_x}\,(\overline{s_{r_1}}\,, \overline{s_{r_2}})\right) = (\acute{r}\,, \underline{s_{\acute{r}}}\,, \overline{s_{\acute{r}}}\,)$.

The next theorem holds the property of an associativity of the intuitionistic fuzzy hyperrings as the following;

- **Theorem**

The intuitionistic fuzzy hyperhomomorphism $\Omega\left(\psi, \underline{\psi_x}\,, \overline{\psi_x}\,\right)$ of intionistic fuzzy hyperrings $\langle(R, I, I), \underline{F}^{+}, \underline{F}^{*}\rangle$ $and, \langle(\acute{R}, I, I), \underline{\tilde{F}}^{+}, \underline{\check{F}}^{*}\rangle.,then$ for all associative intuitionistic fuzzy subhyperrings $\langle S, \underline{F}^{+}, \underline{F}^{*}\,\rangle$ of intuitionistic fuzzy hyperrings under **Ω**, it is mapped to an intuitionistic fuzzy subhyperring Ω (S) of $\langle(\psi(S), I\,, I\,), \underline{F}^{+}, \underline{F}^{*}\,\rangle$.

- **Proof**

If we have $\mathbf{S} = \{(\boldsymbol{r}, \underline{\boldsymbol{s_r}}\,, \overline{\boldsymbol{s_r}}\,) : \boldsymbol{r} \in \boldsymbol{s_0}\}$ be an intuitionistic fuzzy subspace and it has associative intuitionistic fuzzy hyperring $\langle(R, I, I), \underline{F}^{+}, \underline{F}^{*}\,\rangle$, then we get

$\langle\Omega(\mathrm{x})\underline{F}^{+}\,\Omega(\mathrm{y})\rangle\ \underline{F}^{+}\ \ \Omega(\mathrm{z}) = \Omega\left(\langle x\underline{F}^{+}\,y\,\rangle\underline{F}^{+}\mathrm{z}\,\right) = \Omega\left(x\underline{F}^{+}\langle y\,\underline{F}^{+}\mathrm{z}\rangle\,\right) = \Omega(x)\underline{F}^{+}\langle\Omega(y)\underline{F}^{+}\Omega(\mathrm{z})\rangle$, and $\langle\Omega(\mathrm{x})\underline{F}^{*}\ \ \Omega(\mathrm{y})\rangle\ \underline{F}^{*}\ \ \Omega(\mathrm{z}) = \Omega\left(\langle x\underline{F}^{*}\,y\,\rangle\underline{F}^{*}\mathrm{z}\,\right) = \Omega\left(x\underline{F}^{*}\langle y\,\underline{F}^{*}\mathrm{z}\rangle\,\right) = \Omega(x)\underline{F}^{*}\langle\Omega(y)\underline{F}^{*}\Omega(\mathrm{z})\rangle$,

if (*x* or *y*) or *z* belongs to intuitionistic fuzzy space (*H* , *I, I*), then we have an intuitionistic fuzzy hyper-homomorphism $\Omega(x) \in (\psi(H), I, I\,) = (H^{'}, I\,, I\,)$, on the other side, if (*x* or *y*) or *z* belongs to intuitionistic fuzzy subspace **S,** then in this time we can say that the intuitionistic fuzzy element ***x*** of **S** belongs to an intuitionistic fuzzy hyper-homomorphism on **S,** i.e., **Ω**(*x*) ∈ **Ω** (S), where **Ω** (S) is an associative in $\langle(\Omega(R), I, I\,), \underline{F}^{+}, \underline{F}^{*}\,\rangle$ whenever S is an associative in $\langle(R, I, I\,), \underline{F}^{+}, \underline{F}^{*}\,\rangle$.

3. Intuitionistic Fuzzy Hyper-Normal of Sub-Hyperrings Nnder (IFHH)

In this section, we show the notation of intuitionistic fuzzy hyper-normal of sub-hyperrings under intuitionistic fuzzy hyper-homomorphism, and we gave the definition of the kernel and the

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image of (IFHH), as we will introduce some properties as the following;

3.1. Definition [5]

The intuitionistic fuzzy subspace S = $\{(r\ ,\ \underline{s_r}\ ,\overline{s_r}\ ) : r\ \in\ s_0\}$ is intuitionistic fuzzy subhyperring of the intuitionistic fuzzy hyperring $\langle(R\ ,I\ ,I),\underline{F}^+\ ,\underline{F}^*\rangle$,

- **I.** S is closed under the intuitionistic fuzzy binary hyperoperations $\underline{F}^+$ and $\underline{F}^*$,
- **II.** $\langle S\ ,\underline{F}^+\ ,\underline{F}^*\rangle$ satisfies the axioms of the (ordinary) fuzzy hyperring.

In a similar manner to the above theorem, we can obtain the following theorem, which introduces the action of intuitionistic fuzzy normal of sub-hyperrings under the intuitionistic fuzzy hyper-homomorphism IFHH.

- **Theorem**

If we have the intuitionistic fuzzy hyper-homomorphism

$\Omega\left(\psi,\underline{\psi_x}\ ,\overline{\psi_x}\ \right):\langle(R,I,I\ ),\underline{F}^+,\underline{F}^*\rangle\rightarrow\langle\left(\acute{R},I,I\ \right),\underline{\tilde{F}}^+,\underline{\check{F}}^*\rangle =$

$\langle(\psi(R),I,I\ ),\underline{\psi_x(F^+)},\overline{\psi_x}\ \ (\underline{F}^*)\ \rangle$,

then every intuitionistic fuzzy normal of sub-hyperring $\langle S,\underline{F}^+,\underline{F}^*\rangle$ of the intuitionistic fuzzy hyperring $\langle(R,I,I\ ),\underline{F}^+,\underline{F}^*\rangle$ is mapped to an intuitionistic fuzzy normal sub-hyperring $\langle\psi(S),\underline{F}^+,\underline{F}^*\rangle$ of $\langle\left(\acute{R},I,I\ \right),\underline{\tilde{F}}^+,\underline{\check{F}}^*\rangle$ under Ω.

- **Proof**

We can get the proof of this theorem from Theorem [9].

- **Example**

If $\langle(R\ ,I\ ,I),\underline{F}^+\ ,\underline{F}^*\rangle$ is intuitionistic fuzzy hyperring defined as in (Example [1]), and let $=\{(b\ ,\{0,\ 1/2\}\ ,$ {1⁄2,1}) , (a, {0.1}, {0.1})}, then

(i) **-** S is intuitionistic fuzzy subspace of the intuitionistic fuzzy space $(R\ ,I\ ,I)$,

(ii) **-** we must show that $\langle S,\underline{F}^+\ ,\underline{F}^*\rangle$ is intuitionistic fuzzy hyperring under $\underline{F}^+\ ,\underline{F}^*$ as

the following: $(b\ ,\{0,\ 1/2\}$, {1⁄2,1}) $\underline{F}^+$ (a, {0.1} , {0.1}) = ({a, b} , {0 , 1⁄2} , {1⁄2 , 1}) $\subseteq S$,

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$(b, \{0,\ 1/2\}, \{1/2, 1\})$ $\underline{F}^*$ (a, {0.1} , {0.1}) = ({a, b} , {0 , 1} , {0 , 1}) $\subseteq$ S,
hence S under $\underline{F}^+, \underline{F}^*$ is closed. Then from (i), (ii) we have $\langle S, \underline{F}^+, \underline{F}^* \rangle$ is intuitionistic fuzzy subhyperring of the intuitionistic fuzzy hyperring $\langle (R, I, I), \underline{F}^+, \underline{F}^* \rangle$.
We know that notations of the kernel are related to the hyper-homomorphism on hyperrings in the ordinary hyperring theory. Hence, now introduce the kernel of the intuitionistic fuzzy hyper-ring as following;

3.2. Definition

If we have an intuitionistic fuzzy hyper-homomorphism Ω on intuitionistic fuzzy hyperrings defined as $\Omega: \langle (R, I, I), \underline{F}^+, \underline{F}^* \rangle \to \langle (\acute{R}, I, I), \underline{\tilde{F}}^+, \underline{\check{F}}^* \rangle$, then we defined the kernel (*kernel*) and the image (*image*) as;

I. The *kernel* of an intuitionistic fuzzy element (*x*,*I*,*I*) of the intuitionistic fuzzy hyperring $\langle (R, I, I), \underline{F}^+, \underline{F}^* \rangle$ on the intuitionistic fuzzy hyper-homomorphism Ω is
$kernel\ (\Omega) = \{(x, I, I) \epsilon \langle (R, I, I), \underline{F}^+, \underline{F}^* \rangle : \Omega\ (x, \mathrm{I}, \mathrm{I}) = (\acute{e}, I, I)\ \in\ \langle (\acute{R}, I, I), \underline{\tilde{F}}^+, \underline{\check{F}}^* \rangle\}$.

II. The *image* of intuitionistic fuzzy elements (*y*, *I* , *I*) of intuitionistic fuzzy hyperring $\langle (\acute{R}, I, I), \underline{\tilde{F}}^+, \underline{\check{F}}^* \rangle$ on the intuitionistic fuzzy hyper-homomorphism Ω is $image\ (\Omega) = \{(y, I, I) \in \langle (\acute{R}, I, I), \underline{\tilde{F}}^+, \underline{\check{F}}^* \rangle : \Omega\ (x, \mathrm{I}, \mathrm{I})$
$(y, I, I), (x, I, I) \epsilon \langle (R, I, I), \underline{F}^+, \underline{F}^* \rangle \}$

- **Remark**

We will introduce the *kernal*(Ω) of intuitionistic fuzzy hyper-elements of intuitionistic fuzzy hyperrings by *ker*Ω, and the *image* (Ω) of intuitionistic fuzzy hyperelements of intuitionistic fuzzy hyperring by *im*Ω.
In the next, we introduce and show some properties of the *ker*Ω and *im*Ω of intuitionistic fuzzy hyper-elements of intuitionistic fuzzy hyperrings as the following;

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3.3. Theorem

If Ω is an intuitionistic fuzzy hyper-homomorphism of two intuitionistic fuzzy hyperrings $\langle (R,I,I),\underline{F}^{+},\underline{F}^{*}\rangle$ and $\langle (\acute{R},I,I),\underline{\tilde{F}}^{+},\underline{\check{F}}^{*}\rangle$, we have

1) Every intuitionistic fuzzy elements (x,I,I), $(x^{-1},I,I)$and (y,I,I) such that $(x,I,I),(x^{-1},I,I) \in \langle (R,I,I),\underline{F}^{+},\underline{F}^{*}\rangle$ and $(y,I,I) \in \ker\Omega$, satisfy the following:
$$(x,I,I)\ \underline{F}^{+}\ (y,I,I)\ \underline{F}^{+}(x^{-1},I,I) \ \in\ ker\Omega,$$
$$(x,I,I)\ \underline{F}^{*}\ (y,I,I)\underline{F}^{*}\ (x^{-1},I,I) \ \in\ ker\Omega.$$
2) For any $ker\Omega$ is an intuitionistic fuzzy sub-hyperring of the intuitionistic fuzzy hyperring $\langle (R,I,I),\underline{F}^{+},\underline{F}^{*}\rangle$.
3) For any $im\Omega$ is an intuitionistic fuzzy sub-hyperring of the intuitionistic fuzzy hyperring $\langle (\acute{R},I,I),\underline{\tilde{F}}^{+},\underline{\check{F}}^{*}\rangle$.

The intuitionistic fuzzy hyper-homomorphism Ω is one-to-one i*ff* the $ker\Omega$ is the identity in the intuitionistic fuzzy hyperring $\langle (R,I,I),\underline{F}^{+},\underline{F}^{*}\rangle$ as $ker\Omega = \{(e,I,I)\}$.

- **Proof**

1) By using the identity property of the intuitionistic fuzzy hyper- homomorphism between intuitionistic fuzzy elements (x,I,I) , (x^{-1},I,I) and (y,I,I) of the intuitionistic fuzzy hyperring $\langle (R,I,I),\underline{F}^{+},\underline{F}^{*}\rangle$, we can get the following;

$$\begin{aligned}\Omega\left((x,I,I)\underline{F}^{+}\ (y,I,I)\underline{F}^{+}(x^{-1},I,I)\right)\\ &= \Omega(x,I,I)\underline{F}^{+}\Omega(y,I,I)\underline{F}^{+}\Omega(x^{-1},I,I)\\ &= \Omega(x,I,I)\underline{F}^{+}(e,I,I)\underline{F}^{+}\Omega(x^{-1},I,I)\\ &= \Omega\left((x,I,I)\underline{F}^{+}(x^{-1},I,I)\right)\\ &= \Omega\ (e,I,I)\ = (\acute{e},I,I),\end{aligned}$$

$$\begin{aligned}\Omega\left((x,I,I)\underline{F}^{*}\ (y,I,I)\underline{F}^{*}(x^{-1},I,I)\right)\\ &= \Omega(x,I,I)\underline{F}^{*}\Omega(y,I,I)\underline{F}^{*}\Omega(x^{-1},I,I)\\ &= \Omega(x,I,I)\underline{F}^{*}(e,I,I)\underline{F}^{*}\Omega(x^{-1},I,I)\\ &= \Omega\left((x,I,I)\underline{F}^{*}(x^{-1},I,I)\right)\end{aligned}$$

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$= \Omega\,(e, I, I) \;=(\acute{e}, I, I),$

2) By using the definition of the intuitionistic fuzzy hyperring and applying (Theorems [1]), straightforwardly, we can get the requirements in (2) and (3) (mentioned in this theory).

3) ($\Longrightarrow$) Let $(y, I\,, I\,)$ be an element of the $ker\Omega$, and if the intuitionistic fuzzy homomorphism Ω is one-to-one, then by properties of the intuitionistic fuzzy hyperrings and the $ker\Omega$, we have $\Omega\,(y, I, I) = (\acute{e}, I, I)$, and $\Omega\,(e, I, I) = (\acute{e}, I, I)$, then $\Omega\,(y, I, I) = \Omega\,(e, I, I)$ and by using the property of Ω ($\Omega\, is\ one-to-one$), we get $(y, I, I) = (e, I, I)$, hence $ker\Omega = \{(e, I, I)\}$.

($\Longleftarrow$) If $(e\,, I\,, I) \in ker\Omega$, such that (x, I, I), $(y\,, I, I)$ are elements in intuitionistic fuzzy hyperring $\langle (R, I, I\,), \underline{F}^{+}, \underline{F}^{*} \rangle$, $\Omega((x,\ I,\ I\ )) = \Omega((y,\ I,\ I\ ))$,therefore

$\Omega\big((y, I, I)\big)\, \underline{\tilde{F}}^{+} \left(\Omega\big((x, I, I)\big)\right)^{-1} =$

$\Omega\big((y, I, I)\big)\, \underline{\check{F}}^{*} \left(\Omega\big((x, I, I)\big)\right)^{-1} = (\acute{e}\,, I, I)$, then

$\Omega((y, I, I)\ \underline{\tilde{F}}^{+}\ (x^{-1}, I, I) = \Omega((y, I, I)\ \underline{\check{F}}^{*}\ (x^{-1}, I, I) = (\acute{e}\,, I, I)$,where hold $(y, I, I)\ \underline{F}^{+}\, (x^{-1}, I, I) = (y, I, I)\ \underline{F}^{*}\, (x^{-1}, I, I) = (e\,, I, I)$, thas is,$(x, I, I) = (y, I, I)$.

- **Example**

Let Ω : (R, I, I)$\longrightarrow$ (S, I, I), (R,I,I) as in (Example [1]), and the intuitionistic fuzzy hyperoperations are defined as the following:

Table 3. The fuzzy pinery operations

x	$\underline{F}^{+}$	$\underline{F}^{*}$
0	{1.00}	{0,00}
a	{0.85}	{0.05}
b	{0.65}	{0,20}
c	{0.45}	{0.35}

As that, we define the comempership and the conονmempership function as the following:

$\underline{f^{+}}_{xy}(r, s) = \frac{(r*s)}{1-(r*s)}$, $\underline{f^{*}}_{xy}(r, s) = \ \max(r, s)$,

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and, if $S = \{0, x, y, z\}$, were $\Omega(0) = 0$, $\Omega(a) = x$, $\Omega(b) = y$ and $\Omega(c) = z$, *t*hen

Table 4. The comembership and conon-membership functions

R	S	$\underline{f^{+}}_{xy}(r)$	$\underline{f^{*}}_{xy}(r)$	$\underline{f^{+}}_{xy}(s)$	$\underline{f^{*}}_{xy}(s)$
0	*0*	{1.00}	{1.00}	{0,00}	{0,00}
a	*X*	{0.85}	{0.85}	{0.05}	{0.05}
b	*Y*	{0.65}	{0.65}	{0,20}	{0,20}
c	*Z*	{0.45}	{0.45}	{0.35}	{0.35}

$Ker(\Omega) = \{r \in R: \Omega(r) = 0, \underline{f^{+}}_{xy}(r, s) = 1, \underline{f^{*}}_{xy}(r, s) = 0\}$,then $Ker(\Omega) = \{0\}$, $Im(\Omega) = \mathrm{b}$.

- **Remark**

1. In the ordinary case of ring theory, we have the *kernal* between a given ordinary ring R_1, R_2 a homomorphism and define a normal ordinary subring.
2. In fuzzy and intuitionistic fuzzy theory, an assertion cannot be secured in the fuzzy and intuitionistic fuzzy *kernel* of the fuzzy and intuitionistic fuzzy hyper-homomorphism (since the kernel needs to insert the concept of a strong fuzzy and intuitionistic fuzzy *kernel* based on the properties and definition of a kernel and associative fuzzy and intuitionistic fuzzy sub hyperrings).
3. If **Ω** is an intuitionistic fuzzy hyper-homomorphism of two intuitionistic fuzzy hyperrings, then the *ker***Ω** is a strong intuitionistic fuzzy kernel and denoted by (S*kernal*Ω) if the *kernal*Ω is an associative intuitionistic fuzzy sub-hyperring $\langle (R, I, I), \underline{F}^{+}, \underline{F}^{*} \rangle$ of intuitionistic fuzzy space (*R, I, I*), as a generalization of concepts of the concepts of fuzzy and intuitionistic fuzzy groups.
4. The condition in the remark number [14], will ensure that a strong intuitionistic fuzzy kernel of intuitionistic fuzzy hyper-homomorphism will indeed define an intuitionistic fuzzy normal of sub-hyperring.

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5. The strong intuitionistic fuzzy kernel of intuitionistic fuzzy hyper-homomorphism between two intuitionistic fuzzy hyperrings will play an important role when introducing the fundamental theorem of the homomorphism of intuitionistic fuzzy hyperrings in a similar case to the case of ordinary homomorphism of hyperrings.

The next corollary gives the necessary condition for the S*kernal* of a hyper-homomorphism as the following:

- **Corollary**

If **the** *kernal*Ω (an intuitionistic fuzzy kernel of the uniform intuitionistic fuzzy hyper-homomorphism $\boldsymbol{\Omega}\left(\psi, \underline{\psi_x}\ , \overline{\psi_x}\right)$ between the intuitionistic fuzzy hyperrings) is a strong intuitionistic fuzzy kernel (S*kernal*Ω), then the membership functions $\underline{\psi_x}$ and the co-membership functions $\overline{\psi_x}$ hold the conditions of t-norm functions.

4. Conclusion

In this section, we have generalized the concepts of the fuzzy theory based on the fuzzy space, we have introduced some concepts of intuitionistic fuzzy hyperrings as the intuitionistic fuzzy hyper-homomorphism of the intuitionistic fuzzy hyperrings and the kernel and image of the intuitionistic fuzzy hyper-homomorphism. A relation between these concepts based on intuitionistic fuzzy space and the concepts of intuitionistic fuzzy hypergroups of [1], based on intuitionistic fuzzy space and the classical ones of [2], based on universal sets is established.

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